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Advanced Charge Structural Modelling for Solid Rocket Motor Systems

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1. Summary

IM Brimstone is a world leading surface attack missile which has a minimum smoke Elastomer Modified Cast Double Base (EMCDB) propellant rocket motor. This is currently in full scale production for the UK MoD used on the Tornado GR4. During the development stages problems were exposed, discussed further in “Development of IM Brimstone Rocket Motor; An IM, Minimum Smoke, Air-Launched System,” A.Strickland. The investigation required development of more advanced charge structural modelling techniques.

This paper will present the modelling and experimental techniques developed in order to aid design integrity and certification of this type of rocket motor. A description of the challenging environmental and in flight loads will be first discussed including cooldown, pressurisation, cold storage and thermal shock cycling. Then a review of the traditional versus the more advanced charge structural modelling techniques using Finite Element Modelling (FEM) will be given. This will establish some of the key methodologies introduced including the use of nonlinear material models. Focus will then turn towards an overview of the failure criteria used for assessing propellants including stress, strain or energy based methods. Methods used for assessing bondline safety factors will also be presented. In service life assessments will also be highlighted to show the results of a cumulative damage model in order to predict the number of thermal shocks to failure. Experimental validation of the methodologies using trial results can be shown to indicate a good correlation between the predicted and the actual failure temperatures. The cumulative damage results will also show the number of predicted thermal shock cycles to failure which meet the design requirement, thus demonstrating design robustness.

2. Introduction

Solid rocket motors have a propellant charge which is required to meet ballistic performance requirements as well as structural integrity requirements. This requires survival of severe mechanical and thermal loads likely to be seen during the complete service life of the motor. This represents a highly dynamic environment experienced by the materials and predicting their response at various rates, pressures, temperatures and loading histories is a difficult task. For a typical example, a rocket motor would be expected to be first stored at low temperatures for a number of weeks followed by in flight sorties or maneuverers and then ignition with rapid pressurisation. Other loads which a motor can experience include thermal shock cycling and vibration loads.

3. Material Models

Linear viscoelasticity has been used historically to model propellants. A phenomenological representation of linear-viscoelasticity is shown in Figure 1. The model represents a linear or nonlinear spring in series with a number of Maxwell elements. This is also sometimes called the Maxwell-Wiechert model.

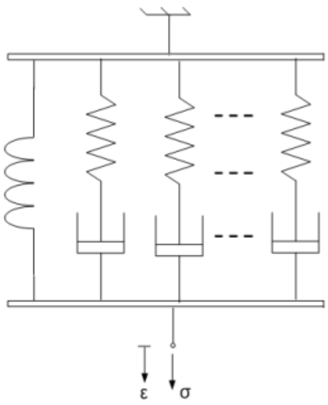

Figure 1: Phenomenological representation of Maxwell-Wiechert Model

The stress in a linear version of the model can be expressed using an alternative form of the hereditary integral formulation using integration by parts it can be shown in [1] that :-

$$\tau(t) = G_0\left(\gamma - \int_0^t \dot{g}_R(s)\gamma(t-s)ds\right) \quad (1)$$

where the time dependent portion and the linear elastic portion are separated. This is a linear form where G_0 is the instantaneous shear modulus. This form can be generalised to yield:-

$$\tau(t) = \tau_0(t) + \int_0^t \dot{g}_R(s)\tau_0(t-s)ds \quad (2)$$

In this form the instantaneous shear is generalised for nonlinear constitutive models as $\tau_0 = \tau_0(\gamma)$ instead of $\tau_0 = G_0\gamma$. The values of $\tau_0(\gamma)$ are calculated based on the hyperelastic model (which determines $G_\infty(\gamma)$) and $g_R(t)$).

The non-dimensional shear relaxation modulus can be represented by a constitutive Prony series of the form:-

$$g_R(t) = 1 - \sum_{i=1}^{N} \bar{g}_i{}^P\left(1 - e^{-\frac{t}{\tau_i{}^G}}\right) \quad (3)$$

where *N* is the Prony series order, while $\bar{g}_i{}^P$ and $\tau_i{}^G$ are the Prony series parameters. The shear relaxation modulus is normalised as follows; $g_R(t) = G_R(t)/G_0$. The parameters used in the modelling were derived from the frequency dependent data. This can be determined from a frequency domain transform of Equation 3. In [1] Temperature effects can be introduced using a reduced time concept which is usually referred to as the thermo-rheologically simple (TRS) temperature dependence

4. Hyperelastic Models

Hyperelastic models have been used for a number of years in elastomer modelling. A few models exist in commercial FE software to describe materials for finite strains i.e. where the strain goes beyond 3%. The models can be used to describe a strain energy potential using the strain invariants:

$$\overline{I_1} = J^{-2/3} I_1 \,;\; I_1 = {\lambda_1}^2 + {\lambda_2}^2 + {\lambda_3}^2 \,; J = \det(\boldsymbol{F}) \quad (4)$$

$$\overline{I_2} = J^{-2/3} I_2 \,; I_2 = {\lambda_1}^2 {\lambda_2}^2 + {\lambda_1}^2 {\lambda_3}^2 + {\lambda_2}^2 {\lambda_3}^2 \quad (5)$$

The Mooney Rivlin is a hyperelastic model sometimes used to describe elastomeric materials with the strain energy function formulated as:-

$$U = C_1(\overline{I}_1 - 3) + C_2(\overline{I}_2 - 3) + \frac{1}{D_1}(J-1)^2 \quad (6)$$

where C_1, C_2 and $D_1$are model parameters. This strain energy potential can be sufficient to model elastomers. However, other more advanced forms exist which include Arruda Boyce, Ogden, van der Waals, etc.

5. User Defined Potentials

With the flexibility of some finite element codes like Abaqus, the user is able to define and implement more advanced strain energy potentials if the choice given above does not fit the data accurately. This has been part of ongoing research to develop models to do this.

6. Propellant Constitutive Response

The elastic moduli of elastomeric propellants depend on time, temperature and hydrostatic pressure. The constitutive response of EMCDB was found to be relatively linear in terms of relaxation spectrum, i.e. rate of relaxation was not found to be dependent on pre-strain level. This was verified by very close correlation between DMTA multi-frequency temperature scan at a pre strain level, as shown in Figure 2. The rate of relaxation may deviate from linearity at higher pre-strains but this has not been thoroughly investigated yet. Nevertheless, assuming a linear viscous response shouldn't lead to significant error and allows for the use of a straightforward description of relaxation using the Prony Series in Abaqus. It was also found that the elastic modulus depended on strain level and was non-linear. This therefore required the use of Hyperelastic models as mentioned earlier to capture the large strain dependence on modulus. Bulk response of EMCDB propellants was investigated using a Farris gas dilatometer. The results showed that at ambient temperature the propellant was nearly incompressible. The experimental investigations led to a conclusion that the propellant response can be captured in Abaqus by either fully or nearly incompressible hyper-viscoelastic constitutive models with a user time-temperature superposition shift function. This approach is advancement on current charge stressing methodology developed in the 1990's.

The Prony Series parameters can be derived from various means either from DMTA Figure 2 or stress relaxation in tension Figure 3. The results from DMTA or tension data can be used to construct a relaxation modulus master curve and a time temperature shift function. Frequency dependent data can also be used for the calibration and an example is shown in Figure 4 with the fitted model. The Prony series was modelled using around 25 parameters, however, more parameters can be used for greater accuracy but this will mean the optimisation function to fit the curves will be more complex and may take a longer time.

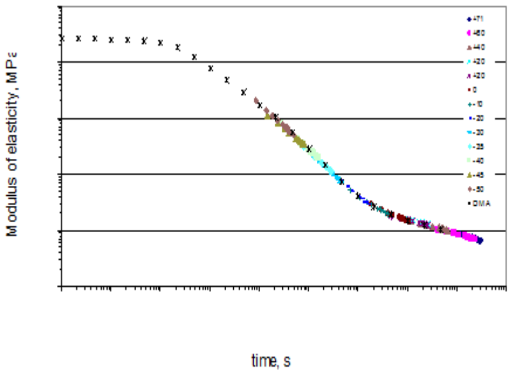

Figure 2: Example Modulus DMTA overlaid by Stress Relaxation Data

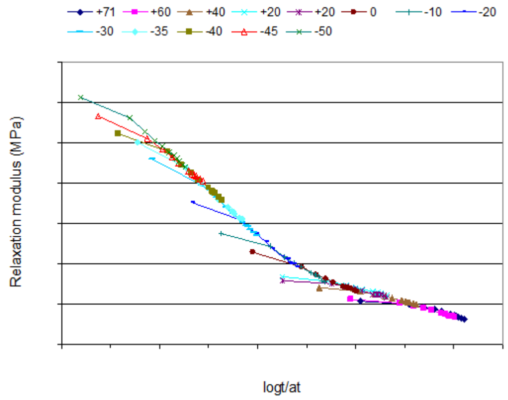

Figure 3: Assembled Relaxation Master Curve

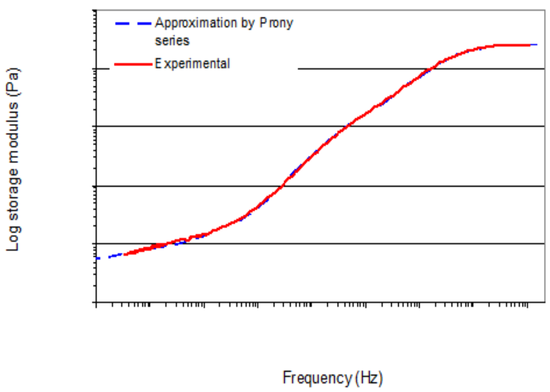

Figure 4: Example Storage Modulus fitted with Prony series

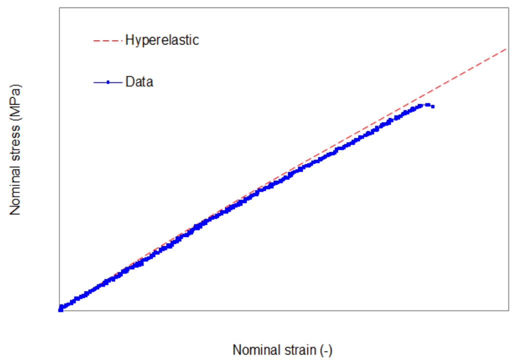

Figure 5: Example Hyperelastic Function Fitted to Model

7. Model Validation

Every constitutive model needs to be verified and validated. Test data other than data used for calibration should be used for validation. Specifically, validation should be carried out for test cases that are relevant to the actual load cases intended for modelling. For example it would make little sense validating hyper-elasticity or hyper-viscoelasticity for compression, if the mode of deformation in the actual load case is predominantly tensile. Typically the validation is carried out against tensile tests performed at various temperatures and strain rates. Modelling needs to be done of the specimens in order to capture the actual strain rate of the material in the gauge. An example validation is shown in Figures 6 and 7 where the data fits the model quite well for the strains being modelled. Figure 8 highlights the improvements in response from the linear to the hyper-viscoelastic models.

Validation using simultaneous thermal-mechanical testing was also conducted where the test specimens underwent straining and cooling at a specified rate close to the actual rates experienced by the propellant in the motor.

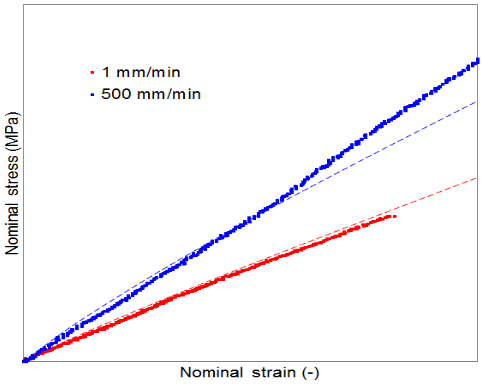

Figure 6: Experimental and Model Predictions for Hot analysis

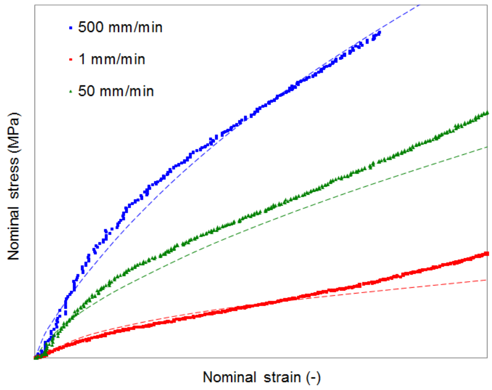

Figure 7: Experimental and Model Predictions for Cold Analysis

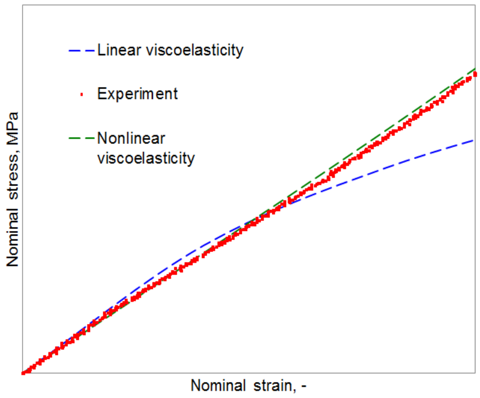

Figure 8: Comparisons between Linear and Nonlinear Viscoelastic Models

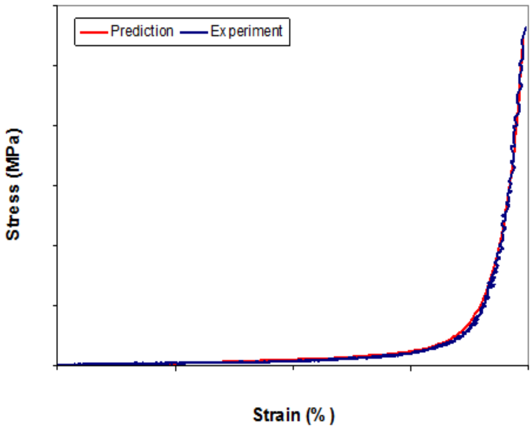

Figure 9: Stress vs Strain during Simultaneous Straining and Cooling

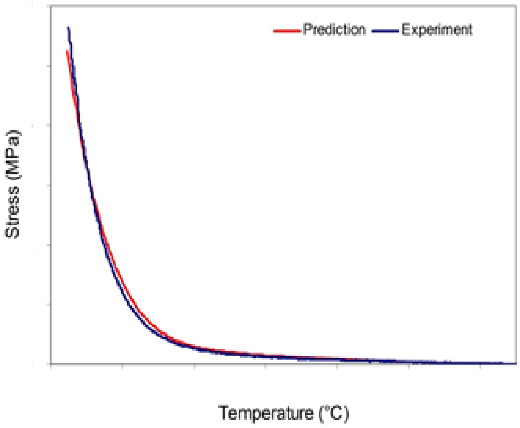

Figure 10: Stress vs Temperature during Simultaneous Straining and Cooling

The simulation of a simultaneous cooling and straining experiment is a very challenging method of model validation as it:

- Requires accurate predictions of stress-strain response over a very wide range of temperatures
- Is sensitive to rate effects and material nonlinearity
- Incorporates contribution from thermal expansion of the material, thus it requires accurate measurements of CTE
- Replicates actual strains and thermal loads experienced on motor cooldown

8. Damage Modelling

Stress softening or Mullins effect is a damage mechanism found in elastomers. This occurs as a gradual breakdown of the mico-structure and is observed as a corresponding reduction in stiffness with loading and unloading. After a few cycles the stiffness stabilises. This was implemented into Abaqus and shows the results as compared with experiment.

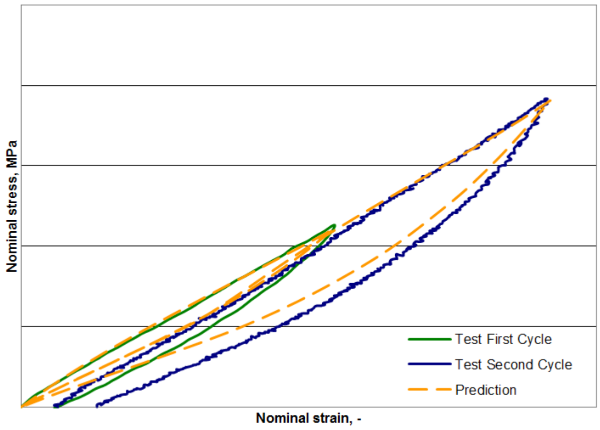

Figure 11: Stress Softening or Mullins Effect Modelling

10. Failure Models

Historically a failure criterion based on strain energy density (SED) was used for prediction of propellant charge cracking during step cooldown and pressurisation. This criterion was successfully validated for the step cooldown trials of various structural test vehicles (STV) motors. The validation work also included stress and strain based failure criteria and it was found that the SED criterion produced results closest to the actual failures.

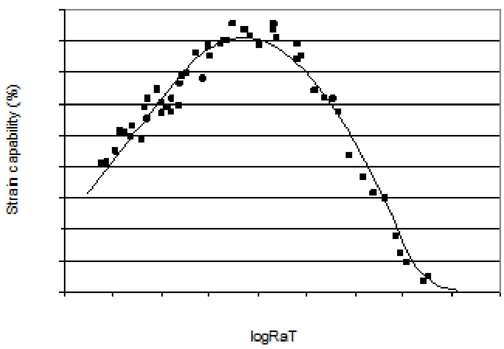

Figure 12: Example Failure Envelope for Strain Capability

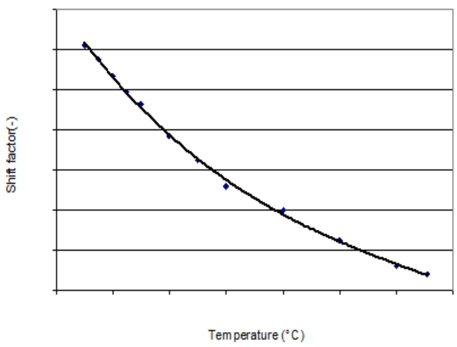

Figure 13: Example Failure Shift Curve

Many different failure criteria's were researched during the Brimstone program. The failure criteria's that were examined were based on the principle that there exists a limit value to stress, strain or the SED that the material can withstand before it fails. This limit value can be called material capability. Hence in the strain energy density failure criteria is expressed by the inequality:- $SED_{load} \geq SED_{capability}$. It is standard engineering practice to evaluate structural integrity of a design by the use of a safety factor which in terms of SED is defined as follows:

$$SF = \frac{SED_{capability}(logRa_T)}{SED_{load}(logRa_T)} \quad (7)$$

Thus a safety factor below 1 constitutes a prediction of failure. The reduced rate is calculated from the following equation:-

$$logRa_T = logR + loga_T(T) \quad (8)$$

where R is usually assumed to be the maximum principal strain rate. $a_T(T)$ is the time temperature equivalent shift factor. The capability and load in the equation for safety factor needs to be given for corresponding reduced rate $logRa_T$. This usually means that the temperature and strain rate during the analysis will be different from the experiment from which the capability data was established.

11. Stassi Stress Criterion

Failure criteria based on SED, von Mises stress or strain do not take into account the effect of the mean normal stress on failure. In general this is acceptable for metals but it was shown that failure of elastomeric materials depends on the hydrostatic pressure, i.e. the mean normal stress in [2]. Hence, for the analysis of failure the propellant charges, hydrostatic pressure should be taken into account. A method which does this is Stassi stress and is given by the following formula:-

$$\sigma_{Stassi} = \frac{\sqrt{9(k-1)^2\sigma_m^2 + 4k\sigma_{eq}^2} - 3(k-1)\sigma_m}{2k} \quad (9)$$

where the equivalent stress σ_{eq} is given by:-

$$\sigma_{eq} = \sqrt{\frac{1}{2}((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)} \quad (10)$$

and the mean stress or hydrostatic pressure is given by:-

$$\sigma_m = -\frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) \quad (11)$$

where $k = {}^{C}/_{T}$ is a ratio between compressive to tensile strength and is a material dependent parameter. For cases where k=1 the result is equal to the von Mises equivalent stress.

This failure method was used as a means for modelling pressurisation. As majority of the other criteria's were found to be inadequate in predicting failure for this load case. This criteria can also be used for the cooldown load case and its results can be shown below in Table 1. The strain safety criteria were found to be too conservative as observed and significantly underestimated the propellant cracking temperature. Equivalent stress failure criterion overestimated charge cracking temperatures. Strain energy density was the failure criterion that provided the closest prediction of charge cracking temperature. Stassi is also a good failure prediction for cooldown.

Method	Failure Temperature
Stress based	-68°C
Strain based	-47°C
SED based	-65°C
Stassi Stress	-60°C
Actual cracking temperature	-62°C to -64°C

Table 1: Predicted Failure Temperatures for Cooldown

13. Calculation of Propellant to Insulation Bond Line Safety Factor

Propellant to insulation bond strength should be determined experimentally by performing tensile tests across a range of temperatures and rates and expressed in terms of logarithmic reduced rate $logRa_T$. The rate can be calculated either as a strain rate or a stress rate. The bondline safety factor is calculated from the following formula:-

$$SF_{bond} = \frac{Bond\ line\ Capability\ (logRa_T)}{Bond\ line\ tensile\ normal\ stress\ (logRa_T)} \quad (12)$$

The bondline normal tensile stress can be obtained from a finite element analysis with contact elements introduced at the propellant to insulation bondline. The compressive stresses at the bondline are ignored here as it is assumed that the bondline will not fail in compression for the loads experienced by the bonded propellant charge. The finite element analysis can be preformed using Abaqus with tied surface to surface contact formulation.

14. Knockdown Factors

Knockdown factors were calculated to account for variability, ageing, thermo-mechanical fatigue (thermal shock cycling) and low temperature strain endurance. The predicted safety factor was then multiplied by knockdown factors to give the cumulative safety factor. To ensure the design meets the requirements the cumulative safety factor has to be at least greater than one. The knockdown factors are given by the following:

$$KDF = \frac{Residual\ (damaged)\ propellant\ capability}{Undamaged\ propellant\ capability} \quad (13)$$

The variability knockdown factor accounts for variability due to charge casting process, property distribution within a single charge, tensile specimen preparation and test variability.

The value of the knockdown factor for ageing was determined from accelerated ageing trial where the propellant was conditioned at 60°C for 3 months, Properties of aged propellant specimens were then evaluated by tensile testing across a range of temperatures and rates. The maximum reduction of mechanical properties was then found at sub-ambient temperatures. Thus, taking the lowest knockdown factor for ageing and applying it across all the shifted rates can be regarded as a conservative approach:-

$$KDF = min\left(\frac{aged\ capability(\log(Ra_t))}{fresh\ capability(\log(Ra_t))}\right) \tag{14}$$

15. Thermal Shock Cycling

The knockdown factor for thermal shock cycling was established based on power law fatigue damage. Two fatigue tensile testing programmes at room temperature were carried out to establish the parameters for the calculation of fatigue damage. The first programme was performed to estimate fatigue life at various load ratios. The second test programme was carried out at a single load ratio and residual properties and post fatigue damage was determined. The parameters determined in the two test programmes were then used in the calculation of the knockdown factor accounting for thermal shock cycling. The knockdown factor was defined by:

$$KDF = \frac{Residual\ capability\ after\ n\ shock\ cycles}{undamaged\ capability} \tag{15}$$

The rocket motors are typically required to survive long periods in cold storage. In order to evaluate the effects of cold storage on the mechanical properties of the propellant, small scale testing was carried out. Tensile specimens were exposed to strain levels similar to the maximum propellant charge strain at a temperature of -46°C for several weeks. Residual properties were then determined by tensile testing and power law parameters established and used to extrapolate damage. The knockdown factor for cold storage strain endurance was then determined:

$$KDF = \frac{Residual\ capability\ after\ n\ days\ cold\ storage}{undamaged\ capability} \tag{16}$$

Predicted Number of Thermal Shock Cycles to Failure		**Motor Test Result**
Average Number	30	>40**
Minimum Number*	17	
Maximum Number*	47	

** Includes coefficient of variation*
*** Test stopped*

Table 2: Predicted number of thermal shock cycles to failure

As can be seen from Table 2 the predicted results are slightly conservative as the rocket motor survived many more thermal shocks. This could be due to variability or other conservatives in the model. Nevertheless this is still a significant advancement in charge modelling and was successfully used to demonstrate Brimstone.

16. Conclusions

A number of conclusions can be drawn from the improvements into the propellant structural analysis methodology:-

- The improved hyper-viscoelastic model provides excellent predictive capability for thermo-mechanical stress-strain response. Better predictions are also made for the isothermal testing than previous material models
- Predictions for charge cracking temperatures are also very close to the experimental observations. This demonstrates the validity of both the material models and failure criteria
- Stassi stress criterion for pressurisation and cooldown was also a significant improvement to the methodologies as all other criteria were inadequate in accounting for hydrostatic pressure
- A model was developed to predict the number of thermal shock cycles to failure of a rocket motor. The predicted number of cycles to failure is good, but was found to be conservative.

17. References

[1] Abaqus 6.10, Analysis User's Manual (2010).Volume 3: Materials , Dassault Systems Simulia.

[2] S. Rabinowitz, I. M. Ward, and J. S. C. Parry, "The effect of hydrostatic pressure on the shear yield behaviour of polymers," *J Mater Sci*, vol. 5, no. 1, pp. 29–39, Jan. 1970.